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A class of two-loop finite $N=1$ SUSY Yang–Mills theories

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Abstract. In this paper, a whole class of two-loop finite $N = 1$ supersymmetric Yang-Mills theories for all groups with the exception of SU(N) is obtained.

Recently, much attention has been paid to finite quantum field theories. Although a large class of $N=4$ finite supersymmetric Yang-Mills (SYM) theories (Sohnuis and West 1981, West 1983, Mandelstam 1983, Brink *et al* 1983) and $N=2$ SYM theories (Howe *et al* 1983, West 1983) has been found, it is not easy, but possible, to construct realistic models based on them (del Aguila *et al* 1985, Dong *et al* 1984) because the representations of the matter multiplets in these theories are real and their Yukawa couplings are too stringent. Then it would be interesting to find a class of finite theories which can be used to construct phenomenologically accepted models: these would be the finite $N=1$ supersymmetric Yang-Mills theories. It has been proved by direct calculation (Parkes and West 1984, West 1984) and by considerations involving the chiral anomaly (Jones and Mezincescu 1984) that some conditions which guarantee one-loop finiteness ensure two-loop finiteness as well. It has not been proved, though, whether these conditions will keep these theories finite to all orders. It is of interest to find all possible two-loop finite theories (Hamidi *et al* 1984) and construct realistic models out of them (Jones and Raby 1984, Hamidi and Schwarz 1984, Dong and Zhou 1985). In this paper, we identify all possible representations of all groups, with the exception of SU(N) (Hamidi *et al* 1984, Jiang and Zhou 1986), which lead to two-loop finiteness.

Consider a globally supersymmetric $N = 1$ theory in four dimensions with a simple gauge group G. It may accommodate matter (chiral) supermultiplets ϕ in an arbitrary representation R of G which contain irreducible representations R^i:

$$R=\bigoplus_i R^i \qquad \phi=\bigoplus_i \phi^i. \tag{1}$$

The superpotential is defined as follows:

$$W=\frac{1}{3!}\sum_{\substack{a,b,c\\ i,j,k}} d^{abc}_{ijk}\phi^i_a\phi^j_b\phi^k_c \tag{2}$$

where the subscripts a, b and c label components of the representations R^i, R^j and exceptional groups only the cubic self-coupling of the adjoint representation possesses

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R^k, respectively. The coefficients d^{abc}_{ijk} can be arranged symmetrically with respect to $\binom{a}{i}$, $\binom{b}{j}$ and $\binom{c}{k}$. The conditions for one-loop finiteness (Parkes and West 1984) are

$$T(R) \equiv \sum_{i=1}^{N} T(R_i) = 3C_2(G) \tag{3}$$

$$\sum_{b,c,j,k} d^{abc}_{ijk} d^{*a'bc}_{i'jk} = 2g^2 \delta_{aa'} \delta_{ii'} C_2(R_i) \tag{4}$$

where $T(R_i)$ and $C_2(R_i)$ are the Dynkin index and the value of the quadratic Casimir operator for the representation R_i, respectively, $C_2(G) = C_2(R_A)$, where R_A is the adjoint representation of G. The anomaly condition is trivially satisfied for all groups with the exception of SU(N).

Our task here is to find all possible solutions for equations (3) and (4). We know that the only irreducible representations R^i that can occur in R are the ones whose indices do not exceed $3C_2(G)$. Singlets are excluded by condition (4). In seeking solutions for equations (3) and (4), it is more convenient, firstly, to consider the following equation:

$$\sum_{\beta,\gamma} P_{\alpha\beta\gamma} = m_\alpha T(R_\alpha) \tag{5}$$

where m_α is the multiplicity of the representation R_α in R. Equation (5) is weaker than equation (4), but it is useful for quickly eliminating many candidates from the list of admissible R.

The procedure for finding all two-loop finite theories for all groups is as follows.

(i) Obtain all solutions of equation (3) for which the index of each irreducible representation does not exceed $3C_2(G)$, as is stated above.

(ii) Use equation (5) to eliminate some of the possible candidates. To do this, one must find all possible Yukawa couplings by using R.

(iii) The remaining candidates are further checked by equation (4).

There are two facts we would like to point out in the last step. The first is that we take a specific set of d_{ijk} such that the right-hand side of equation (4) is diagonal in i and i'. We shall call this the diagonality condition.

Now, from equation (4), we know that we need to calculate

$$A_{ii'} = \sum_{jk} d_{ijk} d^*_{i'jk}.$$

Let $A_{ii'}$ denote a matrix element of the matrix A. Due to the diagonality condition mentioned earlier, equation (4) becomes

$$\sum_{jk} |d_{ijk}|^2 = C_i.$$

Here C_i are constants related to $C_2(R_i)$. Obviously this is a linear equation system under $|d_{ijk}|^2$, so it is very easy to deal with.

The second fact is that for the Yukawa couplings $\uparrow\uparrow\square$, $\square\ \square\ \genfrac{}{}{0pt}{}{\square}{\square}$ and $\genfrac{}{}{0pt}{}{\square}{\square}\ \genfrac{}{}{0pt}{}{\square}{\square}\ \genfrac{}{}{0pt}{}{\square}{\square}$ in the case of SO(N), there is no repeated superfield. In other words, the multiplicity m_α of the representation R^α in R must be larger than two, otherwise the Yukawa coupling vanishes. For example, the Yukawa coupling $\uparrow\uparrow\square$ must be vanishing if there is only one spinor representation $\uparrow$ in R. For the Sp($2N$) case, the Yukawa couplings $\square\square\genfrac{}{}{0pt}{}{\square}{\square}$, $\genfrac{}{}{0pt}{}{\square}{\square}\ \genfrac{}{}{0pt}{}{\square}{\square}\ \square\!\square$ and $\square\!\square\ \square\!\square\ \square\!\square$ possess the same property as stated above, while for the

Table 1. Multiplicities m_α of the irreducible components R_α of R for all the solutions for SO(n).

	Irrep					
	□	↑	□ □	□□	□ □ □	Comments
Dim	7	8	21	27	35	
Index	2	2	10	18	20	
SO(7)			1		1	
	n	$5-n$			1	$n=0, 1, 2$
	$1-n$	n	1	1		$n=0, 1$
	$6-n$	n		1		$n=0, 1, \ldots, 4$
			3			
	$10-2n$	$2n$	1			$n=0, 1, \ldots, 5$
	5	10				
Dim	9	16	36	44	84	
Index	2	4	14	22	42	
SO(9)					1	
	$3-2n$	n	1	1		$n=0, 1$
	$10-2n$	n		1		$n=0, 1, \ldots, 3$
			3			
	$14-2n$	n	1			$n=0, 1, \ldots, 7$
	7	7				

	Irrep					
	□	↑	↓	□ □	□□	Comments
Dim	14	64	64	91	104	
Index	2	16	16	24	32	
SO(14)	4				2	
	$20-12n$			n	1	$n=0, 1$
				3		
	$24-16n$	n	n	1		$n=0, 1$
Dim	$2N$	2^{N-1}	2^{N-1}	$N(2N-1)$	$N(2N+1)-1$	
Index	2	2^{N-3}	2^{N-3}	$4(N-1)$	$4(N+1)$	
SO($2N$)	$2N-10$				2	$N \geqslant 8$
	$2N-6$			1	1	
	$4(N-2)$				1	
	$4(N-1)$			1		
				3		
	12	1		1		$N=8$
	16	1				$N=9$
Dim	10	16	16	45	54	
Index	2	4	4	16	24	
SO(10)	$4-2n$		n	1	1	$n=0, 1$
	$10-2n$	1	n		1	$n=1, 2, 3$

Table 1. (continued)

	Irrep					
	□	↑	↓	□ □	□□	Comments
Dim	10	16	16	45	54	
Index	2	4	4	16	24	
	$12-2n$		n		1	$n=0,1,\ldots,4$
	4	2	2		1	
		1	1	1	1	
					2	
	$16-4n$	n	n	1		$n=0,1,\ldots,4$
				3		
	8	$8-n$	n			$n=0,1,\ldots,4$
Dim	12	32	32	66	77	
Index	2	8	8	20	28	
SO(12)	2				2	
	2	1		1	1	
	$16-10n$			n	1	$n=0,1$
				3		
		n	m	1		$n+m=5$
	4	n	m	1		$n+m=4$
	8	n	m	1		$n+m=3$
	12	n	m	1		$n+m=2$
	$20-4n$	n		1		$n=0,1$

	Irrep							
	□	↑	↓	□ □	□□	□ □ □ □	□ * □ □ □	Comments
Dim	8	8	8	28	35	35	35	
Index	2	2	2	12	20	20	20	
SO(8)	2			1	1			
		2		1	1			
			2	1	1			
	2			1		1		
		2		1		1		
			2	1		1		
	2			1			1	
		2		1			1	
			2	1			1	
	8				1			
	6	1	1		1			
	4	2	2		1			
	8					1		
	6	1	1			1		
	4	2	2			1		
	8						1	
	6	1	1				1	
	4	2	2				1	
				3				

Table 1. (continued)

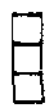
	Irrep							
	□	↑	↓	□ □	□□	□ □ □	□* □ □ □	Comments
Dim	8	8	8	28	35	35	35	
Index	2	2	2	12	20	20	20	
	12			1				
	10	1	1	1				
	8	2	2	1				
	6	3	3	1				
	4	4	4	1				
	2	5	5	1				
	6	6	6					

	Irrep				
	□	↑	□ □	□□	Comments
Dim	11	32	55	65	
Index	2	8	18	26	
SO(11)	1			2	
	$5-4n$	n	1	1	$n=0, 1$
	14			1	
			3		
	$18-4n$	n	1		$n=0, 1, \ldots, 4$
Dim	13	64	78	90	
Index	2	16	22	30	
SO(13)	3			2	
	$18-11n$		n	1	$n=0, 1$
			3		
	$22-8n$	n	1		$n=0, 1, 2$
Dim	$2N+1$	2^N	$N(2N+1)$	$(N+1)(2N+1)-1$	
Index	2	2^{N-2}	$2(2N-1)$	$2(2N+3)$	
SO($2N+1$)	$2N-9$			2	$N \geqslant 7$
	$2N-5$		1	1	
	$4N-6$			1	
	$4N-2$		1		
			3		
	10	1	1		$N=7$

exceptional groups only the cubic self-coupling of the adjoint representation possesses such a property. Therefore we get all possible solutions for all groups, with the exception of SU(N), and they are listed in table 1 for SO(N), table 2 for Sp($2N$) and table 3 for the exceptional groups.

Table 2. Multiplicities m_α of the irreducible components R_α of R for all the solutions for Sp($2N$).

	Irrep					
						Comments
Dim	4	5	10	14	16	
Index	1	2	6	14	12	
Sp(4)	0	0	1	0	1	
	$4-2n$	n	0	1	0	$n=1, 2$
	0	0	3	0	0	
	0	3	2	0	0	
	$12-2n$	n	1	0	0	$n=0, 1, \ldots, 6$
	12	3	0	0	0	

	Irrep				
					Comments
Dim	6	14	21	14	
Index	1	4	8	5	
Sp(6)	0	0	3	0	
	0	2	2	0	
	$16-4n$	n	1	0	$n=0, 1, \ldots, 4$
	$11-4n$	n	1	1	$n=0, 1, 2$
	$6-4n$	n	1	2	$n=0, 1$
	1	0	1	3	
	$24-4n$	n	0	0	$n=2, 3, \ldots, 6$
	$19-4n$	n	0	1	$n=2, 3$

	Irrep					
						Comments
Dim	8	27	36	42	48	
Index	1	6	10	14	14	
Sp(8)	$6-6n$	n	1	0	1	$n=0, 1$
	$6-6n$	n	1	1	0	$n=0, 1$
	$20-6n$	n	1	0	0	$n=0, 1, 2, 3$
	0	0	3	0	0	
	12	3	0	0	0	
	0	5	0	0	0	

	Irrep				
					Comments
Dim	10	44	55	110	
Index	1	8	12	27	
Sp(10)	0	0	3	0	
	$24-8n$	n	1	0	$n=0, 1, 2, 3$
	$36-8n$	n	0	0	$n=2, 3, 4$

Table 2. (continued)

	Irrep			
	□	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	□□	Comments
Dim	$2N$	$N(2N-1)-1$	$N(2N+1)$	$N \geqslant 5$
Index	1	$2N-2$	$2N+2$	
Sp($2N$)	0	0	3	
	$2(2N+2)-(2N-2)m$	m	1	$2(2N+2)/(2N-2) \geqslant m \geqslant 0$
	$3(2N+2)-(2N-2)m$	m	0	$3(2N+2)/(2N-2) \geqslant m \geqslant 2$

Table 3. Multiplicities m_α of the irreducible components R_α of R for all solutions for E_6, E_7, E_8, F_4 and G_2 groups.

	Irrep			
	27	27*	78	Comments
Index	6	6	24	
E_6	0	0	3	
	4	4	1	
	m	n	0	$m+n=$ i2

	Irrep							
	56	133	248	26	52	7	14	27
Index	12	36	60	6	18	2	8	18
E_7	0	3						
	6	1						
E_8			3					
F_4				0	3			
				6	1			
				9	0			
G_2						3	0	1
						8	1	0
						0	3	0

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